

# Analysis of a Microwave-Heated Planar Propagating Hydrogen Plasma

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The heating of a gas to high temperatures by absorption of microwave radiation has been proposed as a potential electrothermal rocket propulsion system. One possible mode of microwave energy absorption is by means of a planar plasma region propagating toward the source of the microwave radiation. Such a planar propagating plasma can be spatially stabilized by a gas stream flowing in the same direction as the microwave radiation with a velocity equal to the plasma propagation velocity. A one-dimensional analysis of the microwave-heated planar propagating plasma for hydrogen gas was developed to predict maximum gas temperatures and propagation velocities. The governing electromagnetic and energy equations were numerically integrated with temperature-dependent thermodynamic properties of equilibrium hydrogen. The propagation velocity eigenvalue was solved by means of an iterative technique. Temperature distribution in the gas, propagation velocities, and percent power absorbed, reflected and transmitted, were obtained as a function of incident microwave power at a frequency of 2.45 GHz for hydrogen gas pressures of 1 and 10 atm.

## Nomenclature

$a$	= long waveguide dimension
$b$	= short waveguide dimension
$c$	= speed of light in a vacuum
$c_1$	= intrinsic impedance
$c_2$	= impedance constant
$C_p$	= specific heat at constant pressure
$E$	= electric field vector
$f$	= frequency
$h$	= enthalpy
$k$	= electromagnetic propagation coefficient
$P$	= power
$t$	= time
$T$	= temperature
$u$	= gas velocity
$U$	= exhaust velocity
$x$	= axial coordinate
$\alpha$	= real component of $k$
$\beta$	= imaginary component of $k$
$\epsilon$	= permittivity
$\lambda$	= thermal conductivity
$\nu$	= collision frequency
$\rho$	= gas density
$\sigma$	= electrical conductivity
$\mu$	= permeability
$\omega$	= circular frequency

## Subscripts

abs	= absorbed
$c$	= cutoff
$e$	= exit static state
$f$	= final state
inc	= incident
IN	= input
$o$	= stagnation or initial state
trans	= transmitted

## Introduction

SEVERAL studies of the use of advanced propulsion concepts for Air Force and NASA missions involving Earth orbit raising and in-orbit maneuvering have been conducted in recent years to identify optimal propulsion schemes.<sup>1,2</sup> The consensus is that, based on present spacecraft electrical power generation capabilities, thrusters with specific impulses between 1000 and 2000 s combine advantages of a powerplant mass lower than that needed for higher specific impulse thrusters and a propellant mass less than that required by chemical propulsion. Unfortunately, this specific impulse range is not well served by current advanced propulsion concepts. Resistojets involving electrical resistance heating of a gas (usually hydrogen) are limited by material constraints to a maximum specific impulse of approximately 1000 s. Arcjet thrusters are able to operate between 1000 and 2000 s because the region of energy addition, the arc, is mostly located away from material walls. But the arc comes in contact with the walls at the electrodes, and electrode erosion along with rejection of waste heat from the electrodes severely limit arcjet lifetimes. Electrostatic ion thrusters and electromagnetic magnetoplasmadynamic (MPD) thrusters both exhibit high specific impulses but suffer efficiency losses when operated below 2000 s. Laser-heated propulsion shows promise of delivering high thrust densities with specific impulses in the region of interest for orbital applications. Laser propulsion, however, has the nonpropulsive problems of target vehicle tracking and high-power laser pointing.<sup>3,4</sup>

The principle of operation of a microwave-heated thruster is shown in Fig. 1. Microwave energy enters the chamber through a dielectric window. The working fluid is injected into the chamber and absorbs the microwave energy in an absorption region which, ideally, would be spatially fixed. The radiation absorption occurs through elastic collisions between the gas molecules and microwave-excited free electrons present in the gas, even at room temperatures. The free electrons are set in motion by the microwave electric field, converting the electromagnetic energy to kinetic energy. The electron kinetic energy is then transferred to the neutral gas and ions by elastic collisions. The high temperature gas is then expanded through a converging-diverging nozzle to obtain thrust by converting the random kinetic energy to directed kinetic energy.

The scheme is identical to that for laser propulsion, but there are several advantages of microwave over laser radia-

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tion. Gas absorptivities are higher at microwave wavelengths than at optical wavelengths, thereby requiring lower power levels and lower gas temperatures for "ignition," a term used to describe the creating of a self-sustaining plasma capable of absorbing the bulk of incident radiation. A second advantage is that microwave wavelengths are of the same order as characteristic chamber dimensions, allowing absorption in resonant microwave structures. Thirdly, state-of-the-art microwave generation is much more efficient than laser generation, approaching 85% efficiency.<sup>5</sup> Therefore, generation of microwave power onboard the spacecraft becomes feasible.

The exhaust velocity is given by an energy balance

$$U_e = \sqrt{2(h_o - h_e)} \quad (1)$$

where  $U_e$  is exhaust velocity,  $h_o$  is chamber stagnation enthalpy, and  $h_e$  is exit static enthalpy. The difference of  $h_o$  and  $h_e$  is amount of energy converted to thrust in the nozzle expansion process. The stagnation enthalpy is proportional to the energy absorbed by the gas while the exit static enthalpy is a function of the expansion process. The enthalpy in terms of the temperature:

$$U_e = \sqrt{2(C_{p_o}T_o - C_{p_e}T_e)} \quad (2)$$

where  $T_o$  and  $T_e$  are stagnation and exit static temperatures, respectively, and  $C_{p_o}$  and  $C_{p_e}$  are specific heats at constant pressure corresponding to chamber stagnation and exit states. Equation (2) enables us to compare various propellants. Hydrogen has the highest value of  $C_p$  and thus has the highest stagnation enthalpy for a given temperature of all candidate propellants, i.e., it has the highest specific impulse for a given chamber stagnation temperature. Due to the high gas temperatures, dissociation and ionization of the propellant gas will occur, resulting in frozen-flow losses in the expansion process. Frozen-flow losses can be kept to a minimum by using high gas pressures in the microwave absorption chamber. The present study, therefore, concentrated on microwave absorption processes that occur for gas pressures of 1 atm or higher.

Beust and Ford<sup>6</sup> were the first to study microwave-heated planar propagating plasmas associated with the arcing observed in high-power transmitters. An air-filled rectangular waveguide  $2.29 \times 1.02$  cm was used, operating with frequencies in the X-band (8.2–12.4 GHz) with power levels up to several kW. The plasma was initiated by inserting a stainless steel screw a small distance into the waveguide. It was observed that the intense heat of the red glowing screw would cause nearby gas to ionize. Following this, small streamer discharges were observed and finally a conducting discharge would form inside the waveguide and propagate toward the microwave energy source. The arc was cylindrical with a diameter of several mm. To prevent damage to the microwave generator a dielectric window was found effective.

Beust and Ford<sup>6</sup> made several experimental observations. Velocity of the discharge was found to increase monotonically with increasing power from 25 cm/s at 300 W to 6 m/s at 2500 W. The power transmitted and reflected were measured. The measurements showed that no microwave radiation was transmitted and that the amount of power reflected was typically 25%. Therefore, by an energy balance, the percentage of power absorbed by the plasma must be approximately 75%. The percentage of power absorbed was found to be dependent upon the power level and waveguide size as well as the temperature and pressure of the gas inside the waveguide.

Batenin et al.<sup>7</sup> experimented with the heavy atomic gases neon, argon, and xenon, using power levels up to 2000 W and pressures up to several atmospheres. The experimental apparatus consisted of a long glass or quartz tube centered

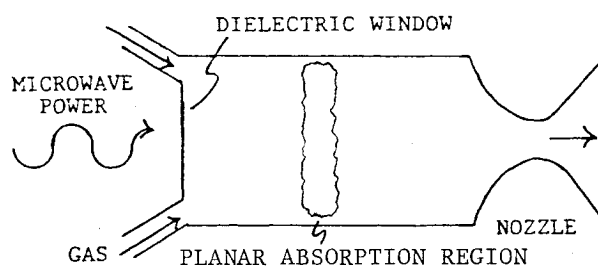


Fig. 1 Principle of operation of a microwave-heated thruster.

inside the waveguide. Batenin et al.<sup>7</sup> found that the heavy inert gases all react quantitatively in the same manner to the microwave radiation. Temperature of the electrons in the discharge was experimentally measured and was found to be between 7000 K and 9000 K, depending little upon input power level, pressure, or tube diameter. The propagation velocity for atmospheric pressure argon at 1.5 kW in a 20 mm diam tube was observed to be approximately 100 m/s. It was found that the significantly higher propagation velocities observed in the inert gases were due to diffusion of resonant radiation from the excited atomic species. In the inert gases the propagation velocities can be significantly reduced with the addition of an impurity, the result of decreased electron temperature due to inelastic collisions and the quenching of the excited inert gas atoms by molecular collisions. Hence, diffusion of the resonant radiation is replaced by the mechanism of atomic thermal conduction; the propagation velocity will, therefore, be considerably lower. An electrostatic probe indicated that 25–30% of the incident power is reflected from the discharge with little dependence on pressure. The remainder of the power was found to be completely absorbed within a distance of 10 cm behind the discharge.

Experiments conducted in atmospheric pressure helium<sup>8</sup> in a rectangular waveguide operating at a frequency of 6 GHz and an input power level of 1–10 kW indicated that the plasma was not in a state of thermal equilibrium. Temperatures of the gas and the electrons were 11,000 K and 27,000 K, respectively. The experiments also indicated that if the helium contained any impurities, the number of excited atoms resonating would be reduced. For example, with a nitrogen impurity of 0.3% at an input power level of 2.6 kW the propagation velocity is only 26 m/s compared to hundreds of m/s for pure helium. It is estimated that even if the nitrogen impurity amounts to only 0.01%, the effect on the mechanism of diffusion of resonant radiation is still significant.

At high radiation flux densities the propagation mechanism of ionizing ultraviolet radiation must be considered. The ultraviolet radiation is produced mainly by the equilibrium discharge plasma. Unless the mechanism of ultraviolet radiation is included at high input power levels, the propagation velocity, as predicted by the atomic thermal conduction mechanism, will underestimate the true velocity. The experimental investigation was carried out by Brodskii et al.<sup>9</sup> using an air-filled circular waveguide with a diameter of 8 cm. The microwave generator operated at a frequency of 85.71 GHz and produced 250 kW of power for a period of 20 ms. Air pressure was atmospheric. At the maximum power of 250 kW the propagation velocity, measured by a collimated photomultiplier, was 100 m/s. At a distance of one cm in front of the ionization wave lies a 1 cm thick area called the pre-plasma region that absorbs 20% of the incident microwave radiation.

Raizer,<sup>10,11</sup> realizing that the mechanism responsible for the propagation in molecular gases was atomic thermal conduction, formulated an idealized one-dimensional, steady-state analysis by establishing an analogy between the propagation of high-pressure discharge waves maintained by microwaves

and the propagation of combustion flames in a closed-end tube. Temperature of the plasma behind the microwave-supported combustion wave and the wave velocity are then functions of gas pressure, absorbed power, and choice of gas. Raizer proceeded to analytically solve the one-dimensional energy and Maxwell equations. Because the temperatures were relatively low, heat losses due to radiation were neglected. Raizer obtained approximate solutions for the maximum gas temperature and propagation velocity as a function of incident microwave power for air and obtained reasonable agreement with the results of Beust and Ford.<sup>6</sup> Myshenkov and Raizer<sup>12</sup> then modeled the propagating microwave plasma where the principle propagation mechanism was the diffusion of resonant radiation. Comparison of their analytical results with experiments conducted with heavy inert gases at very low pressures (3 mm Hg) yielded qualitative agreement.

### Model

Since hydrogen is a molecular gas, the principle process governing the propagation of the microwave-heated plasma is thermal conduction to the unheated gas upstream of the plasma. Because the plasma temperatures are relatively low (<10,000 K), heat loss due to thermal radiation was neglected. The plasma was assumed to be planar and stationary with the cold hydrogen gas flowing into the plasma and the heated hydrogen gas exiting from the plasma. The plasma was assumed to be nonaccelerating and located in a constant area duct. Thus, the governing equations reduce to their steady-state one-dimensional form. The governing equations follow those utilized by Raizer<sup>10,11</sup> for propagating microwave-heated plasmas in air. The present analysis differs from Raizer in that tabulated values of the hydrogen gas properties as a function of temperature are included in this model for results that can be directly compared to future planned experiments.

$$\rho u C_p \frac{dT}{dx} = \frac{d}{dx} \lambda \frac{dT}{dx} + \frac{\sigma}{2} |E|^2 \quad (3)$$

where  $\rho$  is gas density,  $u$  is gas velocity,  $C_p$  is specific heat at constant pressure,  $T$  is gas temperature,  $x$  is axial coordinate,  $\lambda$  is thermal conductivity,  $\sigma$  is electrical conductivity of the gas, and  $E$  is the complex electric field. From the conservation of mass in a constant area duct,

$$\rho u = \rho_o u_o = \text{constant} \quad (4)$$

Thus  $\rho u$  in the energy equation can be replaced by  $\rho_o u_o$ , the conditions upstream of the plasma in the cold gas. The term on the left side of Eq. (3) is a convection term representing the rate of increase in internal energy due to convection inside the control volume per unit volume. The first term on the right side represents the net rate of heat conducted into control volume per unit volume. This term can be expanded by the product rule into two terms, thereby avoiding the assumption of constant thermal conductivity

$$\frac{d}{dx} \lambda \frac{dT}{dx} = \lambda \frac{d^2 T}{dx^2} + \frac{d\lambda}{dT} \left( \frac{dT}{dx} \right)^2 \quad (5)$$

The second term on the right side of Eq. (3) represents rate of energy generated per unit volume inside the control volume by the microwave radiation.

The propagation of planar monochromatic electromagnetic waves through an isotropic medium with a finite electrical conductivity is described by a wave equation

$$\nabla^2 E = \sigma \mu_o \frac{\partial E}{\partial t} + \epsilon \mu_o \frac{\partial^2 E}{\partial t^2} \quad (6)$$

that is derived from Maxwell's equations. Distribution of currents induced by the radiation interacting with the surface of the discharge has been neglected. The permittivity

$$\epsilon = \epsilon_o - \left( \frac{\sigma}{\bar{\nu}_{\text{eff}}} \right) \quad (7)$$

does not remain constant and becomes negative at temperatures greater than 5240 K for atmospheric pressure hydrogen and negative at temperatures greater than 6840 K for hydrogen at a pressure of 10 atm. If the time dependence of the electric field is sinusoidal

$$E(x, t) = E(x) e^{-i\omega t} \quad (8)$$

the equation describing spatial variation of the electric field becomes

$$\frac{d^2 E}{dx^2} + (\omega^2 \mu_o \epsilon + i\omega \sigma \mu_o) E = 0 \quad (9)$$

The solution to Eq. (9) is found by assuming an exponential solution of the form

$$E = E_o e^{-ikx} \quad (10)$$

Substituting Eq. (10) into Eq. (9) and solving the characteristic equation the real component of  $k$  is found to be

$$\alpha = \omega \left\{ \frac{\mu_o \epsilon}{2} \left[ 1 + \sqrt{1 + \left( \frac{\sigma}{\epsilon \omega} \right)^2} \right] \right\}^{1/2} \quad (11)$$

and the imaginary component is

$$\beta = \frac{\mu_o \sigma}{2} \left\{ \frac{\mu_o \epsilon}{2} \left[ 1 + \sqrt{1 + \left( \frac{\sigma}{\epsilon \omega} \right)^2} \right] \right\}^{-1/2} \quad (12)$$

Now that we explicitly know the components of  $k$ , we can determine the electric field as a function of  $x$ . If we start from Eq. (10) and use Euler's identity, the complex electric field becomes

$$E = E_o e^{-\beta x} \cos(\alpha x) - i E_o e^{-\beta x} \sin(\alpha x) \quad (13)$$

that describes an exponentially damped wave. Following this, we square Eq. (13) and take the magnitude to finally obtain the desired result. Simplifying, with the help of some trigonometric identities, magnitude of the square of the electric field becomes

$$|E|^2 = E_o^2 e^{-2\beta x} \quad (14)$$

Because  $\beta$ , which is the imaginary component of  $k$ , is always positive [see Eq. (12)], magnitude of the square of the electric field will always asymptotically approach zero.

The energy equation [Eq. (3)] is solved numerically using a fifth/sixth-order variable step Runge-Kutta scheme. Magnitude of the electric field in the source term is obtained from Eq. (14). The second order energy equation is over-defined mathematically, because in addition to the initial boundary conditions upstream of the plasma

$$T(0) = T_o \quad (15)$$

$$\frac{dT}{dx}(0) = 0 \quad (16)$$

where  $T_o$  is initial temperature of the unheated gas, a third boundary condition downstream of the plasma must also be satisfied

$$\frac{dT}{dx}(\infty) = 0 \quad (17)$$

This additional boundary condition allows us to determine the eigenvalue ( $\rho_o u_o$ ) of the system by an iterative scheme similar to that used by Kemp and Root for the laser-heated plasma wave.<sup>13</sup> Physically, the eigenvalue represents mass flow per unit area. In order to start the iterative process the mass flow is estimated and the integration is performed until two mass flows are obtained, one that is too high and one too low. Whether the mass flow is an upper or lower bound is determined by observing behavior of the first derivative of temperature with respect to  $x$ . If the mass flow is an upper bound, the derivative will either rise indefinitely or reach a maximum, drop toward zero and then rise again. However, if the mass flow is a lower bound, the derivative will reach a maximum and then suddenly drop below zero. The integration is again performed to determine if the average of the bounding eigenvalues behaves as an upper (lower) bound. If this is the case, then the old upper (lower) bound is updated. When the upper and lower bounds have converged upon the same mass flow, within a specified tolerance, the iteration process is stopped.

Once the eigenvalue is determined, velocity of propagation is obtained from

$$u_o = \frac{\rho_o u_o}{\rho_o} \quad (18)$$

and velocity of the discharge with respect to the heated gas

$$u_f = \frac{\rho_o u_o}{\rho_f} \quad (19)$$

where subscript  $f$  refers to final state of the plasma. For a microwave-heated plasma propagating from the closed end of a duct, it is velocity of discharge with respect to the heated gas, not the propagation velocity, which should agree with the experimentally observed values since the closed end forces the velocity of the heated gas to be zero.

The initial boundary condition, stating that the first derivative of temperature with respect to the distance from the power source is approximately equal to zero [Eq. (16)], requires further clarification. For example, if a value of zero is chosen, temperature does not increase with the distance from power source as is to be expected, but remains constant at its initial value (i.e., the trivial solution). The physically realistic solution for the temperature profile can be found by assuming a value slightly greater than zero. However, if this value is substantially greater than zero, we have altered the initial boundary condition. Therefore, a preliminary examination was conducted to determine an acceptable intermediate value for this initial boundary condition without either obtaining the trivial solution or altering the problem. The initial value for the first derivative of temperature with respect to the distance from the power source was selected to be 0.0001 K/m. It was noted that further reduction of this value did not result in any substantial variations in the propagation velocity of the ionization wave or the maximum plasma temperature.

Since the governing equations were solved numerically, tabulated values of the equilibrium hydrogen gas properties as a function of temperature for gas pressures of 1 and 10 atm were incorporated into the numerical code. Specific heat at constant pressure between 200 K and 8000 K was obtained from Patch.<sup>14</sup> Thermal conductivity between 200 K and 8000

K was obtained from Touloukian et al.<sup>15</sup> and Vargaftik and Vasilevskaya.<sup>16</sup> Effective collision frequency for use in Eq. (7) was obtained from Itikawa<sup>17</sup> and electrical conductivity up to 10,000 K was obtained from Yos.<sup>18</sup> Due to the high hydrogen gas pressures examined in this study,  $v_{\text{eff}}^2 \gg \omega^2$ , and only the dc component of the electrical conductivity was required. A cubic spline was used to interpolate between the tabulated values of the above gas properties as a function of temperature.

In order to compare the analytical results to future experiments (no previous experiments were conducted with hydrogen) the maximum electric field intensity as a function of incident microwave power in a waveguide is obtained from

$$E_o = \sqrt{\frac{4c_1 P_{\text{IN}}}{abc_2}} \quad (20)$$

where  $a$  and  $b$  are dimensions of the waveguide and  $P_{\text{IN}}$  is the input power.<sup>19</sup> Intrinsic impedance ( $c_1$ ) and another constant ( $c_2$ ) are

$$c_1 = \sqrt{\frac{\mu_o}{\epsilon}} \quad (21)$$

$$c_2 = \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (22)$$

where  $f$  is frequency and  $f_c$  is cutoff frequency

$$f_c = \frac{c}{2a} \quad (23)$$

where  $c$  is speed of light in a vacuum.

The microwave power in the waveguide before it encounters the ionizing plasma is referred to as the power incident at the plasma

$$P_{\text{inc}} = \frac{1}{2} \frac{E_o^2}{c_1} \quad (24)$$

The amount of power transmitted is known once the wave equation is solved for the value of the electric field at the final state

$$P_{\text{trans}} = \frac{1}{2} \frac{E_f^2}{c_1} \quad (25)$$

When the radiation interacts with the plasma, frequency and amplitude of the microwave will decrease and the plasma will absorb a percentage of the electromagnetic energy depending upon temperature of the plasma and depth of penetration of the microwave into the plasma

$$P_{\text{abs}} = \frac{1}{2} \int_0^f \sigma |E^2| dx \quad (26)$$

The amount of power reflected was determined by subtracting the absorbed and transmitted powers from the incident power. This can be verified by a separate calculation of the reflectivity of the planar plasma once plasma temperature, and therefore the electrical conductivity, is known.

## Results

The numerical calculations were performed for hydrogen at pressures of 1 and 10 atm at a frequency of 2.45 GHz in a rectangular waveguide 7.214 × 3.404 cm. Input power level

was varied from 250 to 3000 W and initial temperature of the unheated gas was 300 K. The degree of accuracy or number of digits to which the eigenvalues must be known for the boundary condition at infinity to be satisfied is directly proportional to the input power level. At a power level of 500 W only six digits of accuracy are required. However, if input power is increased to 3000 W, then the eigenvalue must be known to sixteen places. If the power level is greater than 3 kW, at a pressure of 1 atm, more than 16 digits are required. Since the program was written in double precision that only retains 16 digits, solutions above an input power of 3000 W were not obtainable. It should be noted, however, that the high degree of precision in computing the eigenvalue is required only to obtain solutions of the temperature profile satisfying Eq. (17), due to the strong influence of the source term in the energy equation; and, the iterative process for solving for the eigenvalue quickly bounds the eigenvalue to the few significant digits needed for comparison with experimental results.

The numerical results are listed in Table 1 for a pressure of 1 atm and in Table 2 for a pressure of 10 atm. Figure 2 plots propagation velocity of the microwave-heated plasma as a function of input power at pressures of 1 and 10 atm. It can be seen that propagation velocity rises as the incident microwave power is increased. The lower propagation velocities at the higher pressure is mainly due to increased gas density even though Table 2 shows that the propagation eigenvalue,  $\rho u$ , is slightly larger for the higher pressure. Figure 3 plots the maximum microwave-heated plasma temperature as a function of input power. It was found that slightly higher plasma temperatures were obtainable with increased gas pressure. Maximum gas temperature increases only slightly with increased incident microwave power because significant dissociation occurs in this temperature range that consumes the increased absorbed power. The main effect of increased incident microwave power, however, is to increase the propagation velocity, thereby causing a larger mass flow of gas to be heated. Assuming frozen flow, these temperatures correspond to a vacuum-specific impulse of approximately 1000 s when expanded through a converging-diverging nozzle. Figure 4 plots the percentage of incident microwave power reflected and absorbed as functions of input power. The percentage of incident microwave power absorbed range from 18% to 31%. Percentage of incident microwave power absorbed decreases with increasing input power but is approximately the same for the two gas pressures. Percentage of power transmitted was found to be zero in all cases examined. Figure 5 shows the temperature profile in the microwave-heated plasma as a function of input power at a pressure of 1 atm. As shown previously in Fig. 3, the maximum plasma temperature in-

creases along with a steepening of the temperature profile as input power increases. Figure 6 shows similar results for a gas pressure of 10 atm.

Although no previous experiments or analyses have been conducted for hydrogen, qualitative comparisons can be made. The percent power absorbed calculated in this study was about half that measured by Beust and Ford<sup>6</sup> for air. Taking into account the reduced percent power absorbed, the calculated propagation velocities and maximum temperature as a function of input microwave power are only slightly less than those measured by Beust and Ford<sup>6</sup> or calculated by Raizer<sup>10,11</sup> for air.

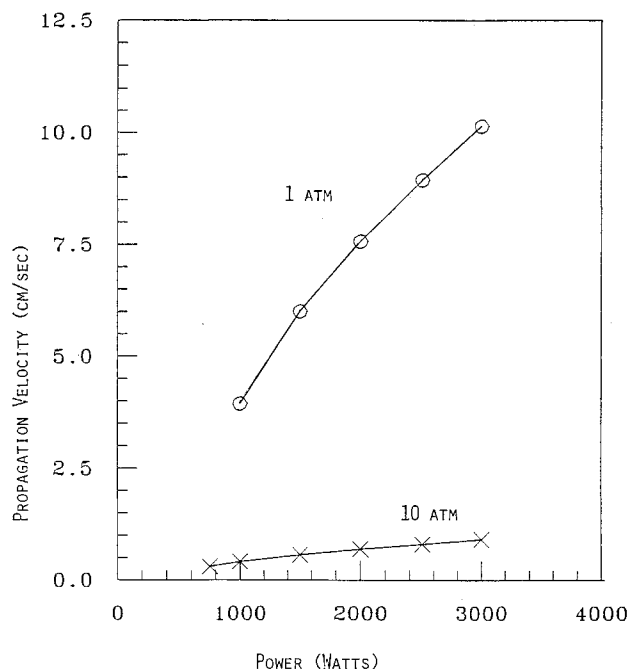


Fig. 2 Propagation velocity of microwave-heated hydrogen plasma as a function of input power showing velocity rise with increased power.

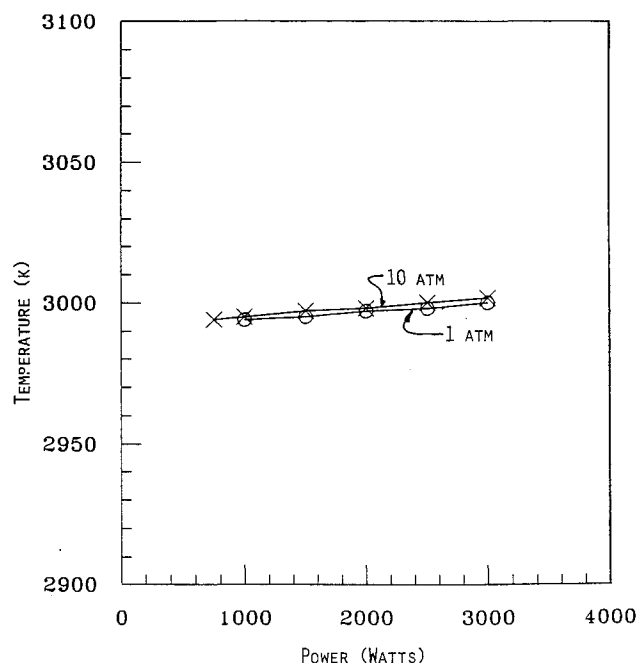


Fig. 3 Maximum microwave-heated plasma temperature as a function of input power showing slightly increased temperature with increased power.

Table 1 Results of hydrogen at 1 atm pressure

$P_{in}$ (W)	$\rho u$ ( $\text{kg}/\text{m}^2 \cdot \text{s}$ )	$u$ (cm/s)	$T_{MAX}$ (K)	% $P_{abs}$
1000	$3.238 \times 10^{-3}$	3.93	2994	30.98
1500	$4.934 \times 10^{-3}$	5.99	2995	25.85
2000	$6.242 \times 10^{-3}$	7.57	2997	26.95
2500	$7.361 \times 10^{-3}$	8.93	2998	17.69
3000	$8.359 \times 10^{-3}$	10.14	3000	18.41

Table 2 Results of hydrogen at 10 atm pressure

$P_{in}$ (W)	$\rho u$ ( $\text{kg}/\text{m}^2 \cdot \text{s}$ )	$u$ (cm/s)	$T_{MAX}$ (K)	% $P_{abs}$
750	$2.499 \times 10^{-3}$	0.303	2994	26.70
1000	$3.358 \times 10^{-3}$	0.407	2995	27.15
1500	$4.687 \times 10^{-3}$	0.569	2997	27.8
2000	$5.766 \times 10^{-3}$	0.696	2998	25.16
2500	$6.706 \times 10^{-3}$	0.814	3000	19.44
3000	$7.549 \times 10^{-3}$	0.916	3002	17.32

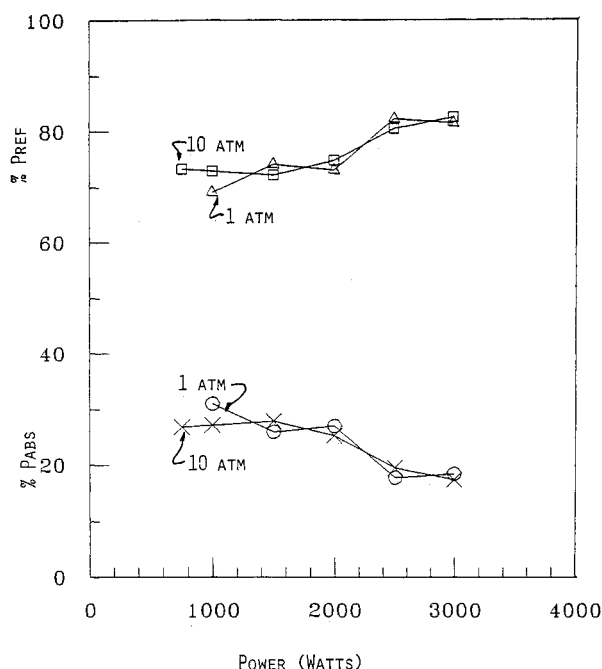


Fig. 4 Percentages of incident microwave power reflected and absorbed as a function of input power. Percentage of power absorbed decreases with increased input power.

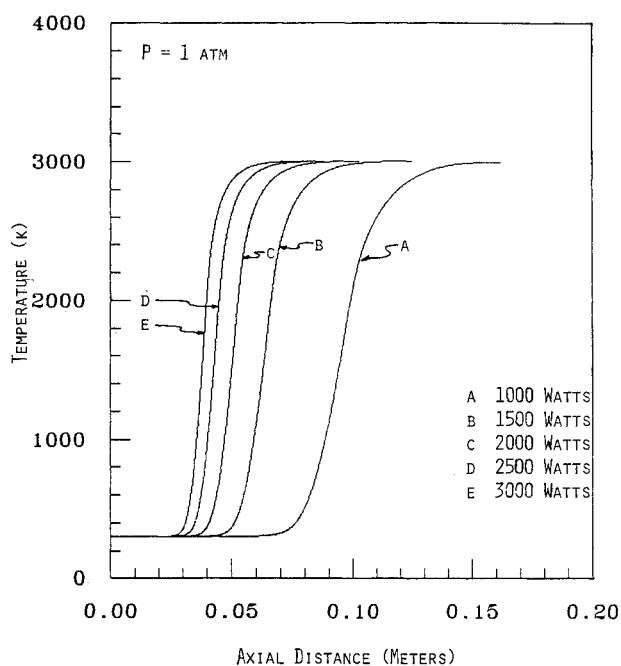


Fig. 5 Temperature profile in the microwave-heated plasma as a function of downstream distance for five values of the input power at 1 atm pressure.

### Conclusions

A numerical model of the microwave-heated planar propagating plasma for hydrogen was developed. Variable gas properties as a function of temperature were incorporated into the code and the propagation mass flux eigenvalue was solved for iteratively. Propagation velocities, maximum plasma temperatures, and percent powers absorbed and reflected at hydrogen gas pressures of 1 and 10 atm were obtained as functions of input microwave power. Propagation velocities and maximum plasma temperatures were found to rise with increased microwave-input power although the

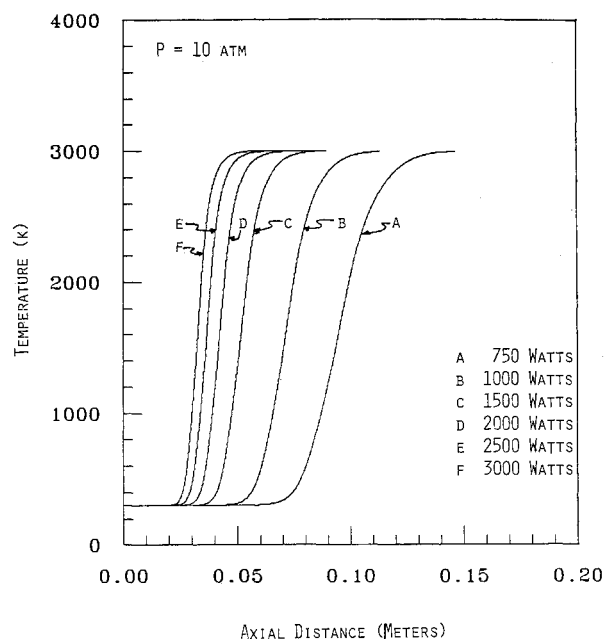


Fig. 6 Temperature profile in the microwave-heated plasma as a function of downstream distance for six values of the input power at 10 atm pressure.

temperature increase with power was not significant. Percent powers absorbed were calculated to be between 18% and 31% with the remaining microwave power reflected. Finally, temperature profiles in the microwave-heated plasma as a function of input power were calculated. Comparisons to previous experimental and analytical results for microwave-heated propagating plasmas in air yielded qualitative agreement.

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